Mapping free software used to teach measurement and proportion

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ABSTRACT
Educational software has significantly changed how mathematics is taught and learned. One of the challenges educators face is choosing software that best meets their teaching goals. Therefore, we mapped free educational software regarding quantities and measures content for mathematics education. The study was conducted with teachers at public middle schools. Various software options were presented to the teachers who then filled out questionnaires on the content focus of the software. Of the thirty-three software packages presented to the teachers, 42% could contribute to measuring skills and proportional reasoning and 30% were appropriate for Numbers and Operations and Space and Shape content. Most of the software packages were developed for geometry. The teachers found that there were several packages that addressed the same content, therefore they could choose the best option for the specific technical requirements of their laboratories, schools and teaching-learning needs for mathematics education.

RESUMO
Os *software* educativos são recursos didáticos que vêm provocando importantes mudanças na forma de ensinar e aprender matemática. Um dos desafios dos educadores é escolher, dentre tantos *software*, o que atenda aos objetivos de sua prática docente. O objetivo deste estudo foi realizar um mapeamento de *software* educativos gratuitos de acordo com os conteúdos de grandezas e medidas no ensino de matemática. O estudo foi realizado com professores do ensino fundamental II de escolas públicas. Foi feita a apresentação das funcionalidades de cada *software* para os professores, os quais preencheram um questionário da relação *software* e conteúdo. Os resultados indicaram que, dos trinta e três *software* apresentados aos professores, 42% podem contribuir para a competência métrica e o raciocínio proporcional. Pela possibilidade da articulação com os outros blocos temáticos, 30% também atendem aos conteúdos de Números e Operações e Espaço e Forma. A maioria destes *software* foi desenvolvida para a geometria. Os professores encontram vários *software* para se trabalhar um mesmo conteúdo, ficando a escolha do professor em função dos requisitos técnicos do laboratório da escola e dos requisitos didático-pedagógicos da educação matemática.

Keywords
Matemática, grandezas e medidas, ensino fundamental.

1. INTRODUCTION
Measurement is necessary in many daily activities. Given this significance, measurements of length, mass, capacity, surface, time, temperature, angles and volume should be important topics in mathematics curricula.

The Brazilian national curriculum parameters - PCNs (Parâmetros Curriculares Nacionais) [Brasil 1998] stipulate that measurement skills should be developed from grades six to nine by creating situations that help students construct and broaden concepts of measurement. These concepts should be developed by studying how various measures are used in daily life and by analyzing historical problems that led to the development of these measures. Recommended methodologies include solving problems involving various measurements and selecting the units of measure and equipment needed to achieve required levels of precision.

Given the PCNs guidelines regarding the teaching of quantities and measures in mathematics, all proposed activities should employ concrete materials [Angeli and Nogueira 2007; Carminati 2008; Versa and Sousa 2008] and software [Dullius and Quartieri, 2010; Sousa 2008; Versa and Sousa 2008].

Several software packages have been developed specifically for education. However, the difficulty for teachers to easily discover and choose among the various options creates a barrier between teaching and these educational resources.

According to Figueira-Sampaio et al. [2012], most teachers choose educational software based on the recommendations of other teachers. In order to list software options and help teachers select viable options for developing the concepts and procedures related to measurement and proportions, we mapped free educational software based on quantities and measurement content.

2. METHODOLOGY
We evaluated 33 software packages proposed by Nogueira et al. [2013] or mentioned in digital or print format. All software packages were free, available for installation and targeted at 6^{th} to 9^{th} grade mathematics instruction.

The mathematical content of each software package was identified and classified according to PCN categories by 34 public school teachers (6th – 9th grade). The features of each software package were demonstrated to the teachers so that they could provide evaluations and suggestions from a didactical-mathematical point of view. The teachers were provided with forms to record the specific math content and educational objectives addressed by each software package.

RESULTS

According to the teachers and considering the thematic blocks stipulated by the PCNs, 42% of the software could be used to teach Quantities and Measurements. Thirty percent of these could also be extended to content in the Numbers and Operations and Space and Shape thematic blocks. Some of these software packages were developed for geometry (e.g. *C.a.R – Regua e Compasso, GeoGebra, Geonext* and *Wingeom*) (Figure 1). These programs can be used to both visualize and construct geometric shapes.

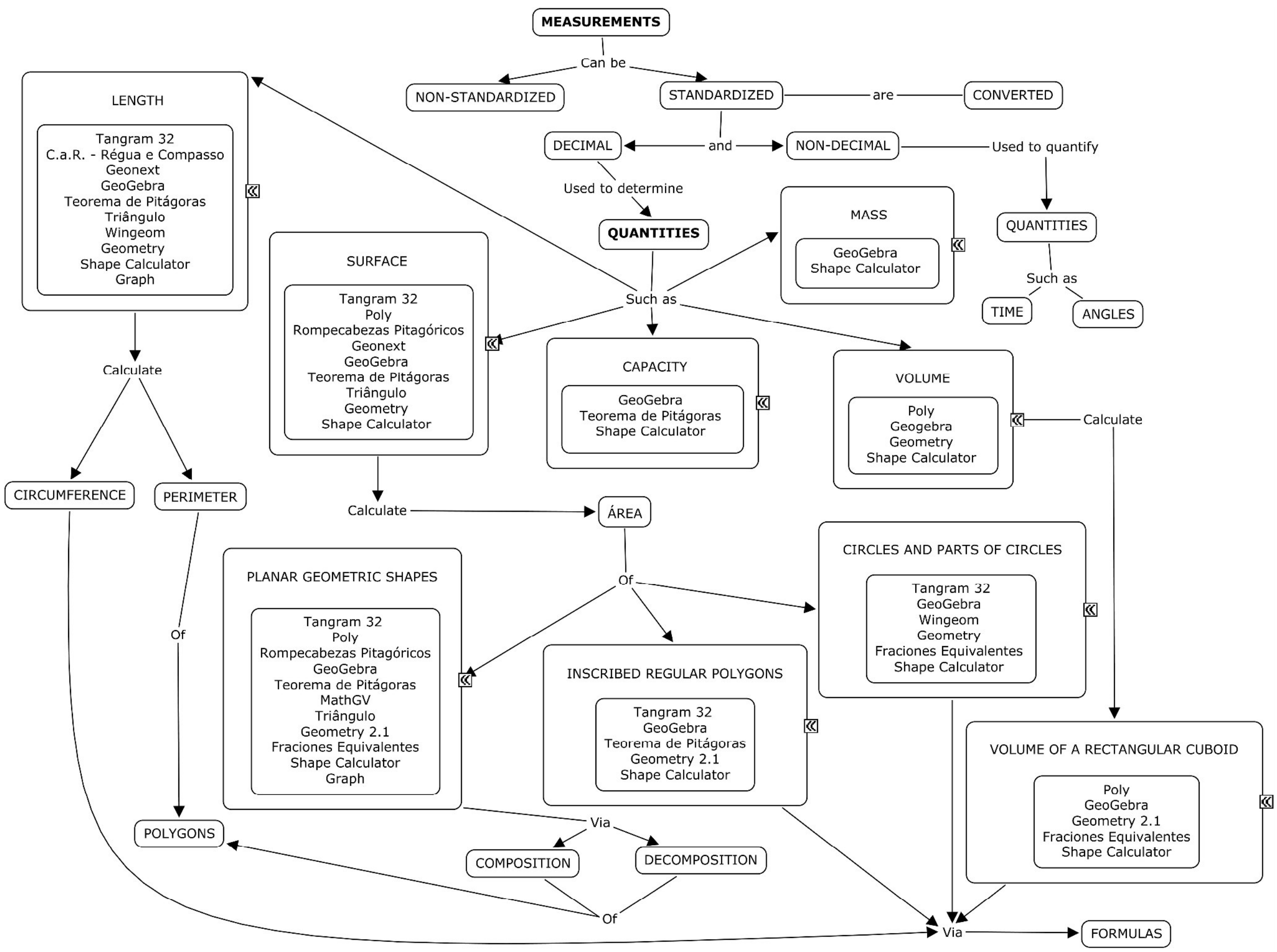

Figure 1. Software mapping by mathematical content.

The teachers identified that *GeoGebra*, *Geonext* and *Wingeom* are primarily focused on length measurements. *GeoGebra* and *Geonext* also included content related to the circle and parts of a circle. The teachers determined that *GeoGebra* in particular was appropriate for surface, volume, capacity and mass measurements, volumes of rectangular cuboids, areas of planar geometric shapes and inscribed regular polygons.

Rompecabezas Pitagóricos, Shape Calculator, Geometry 2.1 and *Tangram 32* can be used to work with surface measurements and planar geometric shapes. According to the teachers, *Tangram 32, Geometry 2.1* and *Shape Calculator* can also be used to work with length measurements, areas of circles and areas of inscribed regular polygons. *Geometry 2.1* and *Shape Calculator* could potentially be used for volume, volume measurements and rectangular cuboids. Finally, *Shape Calculator* could also be used to work with measurements of capacity and mass.

Our results showed that *C.a.R - Régua e Compasso* and *MathGV* were the software packages that least contributed to the development of measurement and proportion skills. The teachers indicated that *C.a.R - Régua e Compasso* could be used to work with length measurements and *MathGV* could be used with areas of planar geometric shapes.

Nineteen teachers indicated that the *Teorema de Pitagoras* package could be used in math education, but only 16% of these teachers thought the software was an appropriate didactic resource for the Quantities and Measures block. Similarly, eighteen and twenty-eight teachers identified the *Triangulo* and *Poly* packages, respectively, as appropriate for math education, while 56% and 18% of these teachers classified the software as useful for quantities.

These three software packages could be used to explore surface measurements and areas of planar geometric shapes, whereas *Teorema de Pitagoras* and *Triangulo* could be used for length measurements.

The *Teorema de Pitagoras* package can also be used with measurements of capacity and areas of inscribed regular polygons. *Poly* targets planar shapes of three-dimensional figures and therefore could also be used to work with surface measurements by only considering the two-dimensional faces of the solids. This software could also be used to work with ideas related to the space occupied by geometric solids and volume.

Elementary students have obvious difficulties in understanding the concept of the area of planar shapes [Lamas et al. 2007]. Common problems with measurements include choosing inappropriate units of measure and solving problems with erroneous and unreal data [Huertas 2009]. Specifically, students frequently confuse perimeter and area when determining the area of a surface [Huertas 2009, Henriques 2013]. Several factors contribute to this problem including using only standard shapes, not understanding the relationship between the perimeter and area of the same shape, not transforming figures to show preservation or modification of area and perimeter [D'Amore and Fandiño Pinilla 2006]. Another difficulty arises from only determining surface area by using formulas that consider only the sides of the geometric shapes [Brito et al. 2009].

Some of the software packages that were identified for surface calculations (Figure 1) also have functions that could be used to correct this misconception between area and perimeter.

For example, GeoGebra allows teachers to develop activities that explore the relationship between the area and perimeter of the same shape. To accomplish this, students first need to construct geometric shapes. To obtain the perimeter and area of the shape, the students need only select desired options and click inside the polygon to visualize the calculation. Kusiak et al. [2012] proposed educational activities where elementary students constructed various geometric shapes and then calculated their perimeter and area. Constructing and manipulating shapes sparked student interest in understanding the relevant concepts of quantity.

Given that formulas are commonly used to teach measurements of quantity [Machin et al. 2003], *Shape Calculator* and *Geometry 2.1* can be used to teach students how to use algebraic expressions to calculate the areas of planar shapes.

The interface of this software shows an algebraic expression and a related geometric shape. The ability to visualize shapes in conjunction with associated expressions and calculations can help students better understand the targeted concepts of quantity. After clicking on a desired option, students are presented with blank text boxes where values can be inserted to calculate volume, surface area and perimeter.

While working with calculations of the perimeter and area of planar geometric shapes and the volume and surface area of geometric solids (Figures 2 and 3), software packages allow teachers to create educational activities that require students to simultaneously use other software or educational resources to determine the necessary measurements for each calculation or geometric shape. These software packages allow navigation through various open windows during the same activity to help students compare calculations or use values from one window to another.

Geometry is an important part of the mathematics curriculum and contributes to learning numbers and measures [Brasil 1998]. The Pythagorean Theorem is taught in geometry classes [Pais 2006] and frequently demonstrated by comparing the areas of planar shapes [Gandulfo et al. 2007; Ribas and Mathias 2012]. This approach allows teachers to work with various concepts within the same activity.

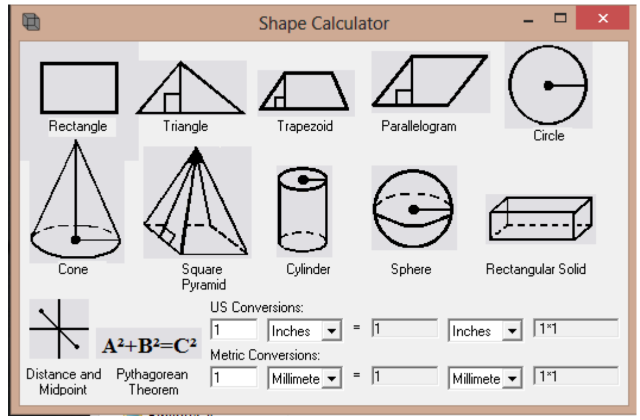

Figure 2. ***Shape Calculator*** **interface.**

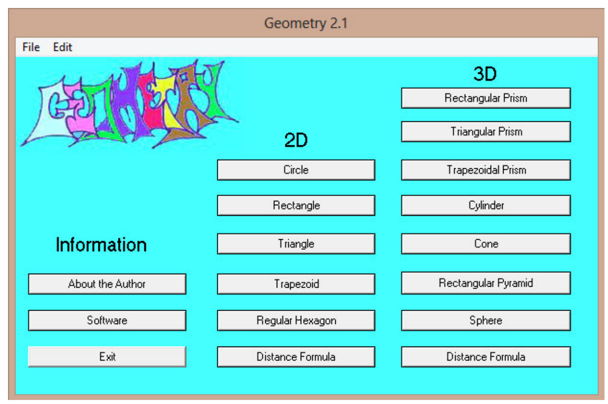

Figure 3. ***Geometry 2.1*** **interface.**

The *Teorema de Pitagoras* software was developed to teach the Pythagorean Theorem and provides exercises at various difficulty levels. These exercises allow students to assess their knowledge and teachers to discuss the theorem and related concepts. Concepts of area and perimeter also appear in the software.

Students have trouble calculating the surface area of irregular shapes or shapes that are not composed of right angles [Huertas 2009]. One solution is to decompose these irregular shapes into parts that are regular and whose surface areas can be more easily calculated and then summed to find the total surface area [Brasil 1998; Scartazzini et al. 2005].

According to the teachers, only *GeoGebra* adequately allowed students to calculate areas of irregular shapes and build irregular polygons with a specified number of sides. *GeoGebra* has been used in investigative activities [Machado 2011; Gomez 2013] to calculate the surface area of irregular shapes by decomposing triangles into quadrilaterals.

The concept of the unit of measure can be developed by starting with qualitative measurements where simple comparisons lead to measurements with non-decimal units (objects) so that students learn systemization [Pontes 1996]. Understanding area and volume can be developed by decomposition or equivalent shapes [Machín et al. 2003].

Building geometric shapes with Tangram puzzle pieces allows students to explore equivalence and decomposition, which in turn helps the students in activities involving area. These individual pieces can be used as different units of measure [Breda et al. 2011]. The *Tangram 32* software is a virtual representation of traditional Tangrams (Figure 4) - a game that challenges players to position seven pieces to form specific shapes. Players are rewarded for minimizing how many times pieces are rotated or moved.

Volume and volume calculations can be explored by unfolding solids into planar shapes [Carminati, 2008; Rocha and Silva 2012]. *Poly* provides a simple interface, a collection of polyhedra and the ability to explore solids by rotating, moving and unfolding (Figure 5). According to Sousa (2008), this software focusses on steady, gradual learning with clear, objective approaches that promote learning. Nicolini et al. [2011] believes that the concept of volume is constructed by analyzing and making deductions while manipulating objects.

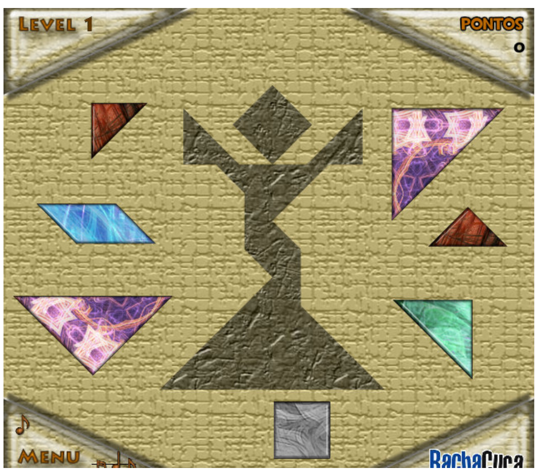

Figure 4. *Tangram 32* interface.

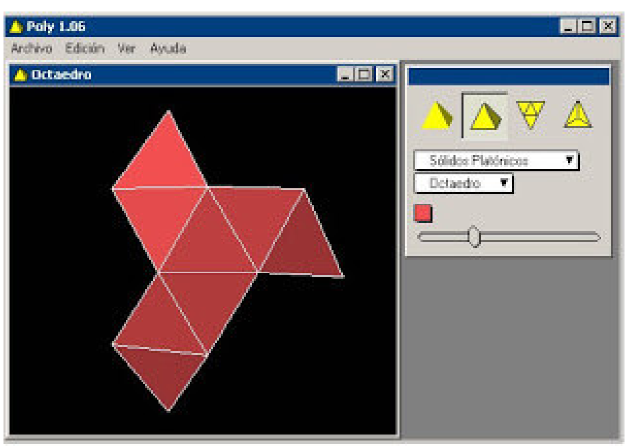

Figure 5. *Poly* interface.

The teachers identified five software packages that could be used to study polygons inscribed in circles (Figure 1). Relationships between measurements can be established by measuring the sides and apothem of the polygon and the radius of the circle. Among 14 teachers, 57% would use *GeoGebra* to explore this content.

To create a container with the minimum amount of material and combining trigonometry, spatial geometry and other mathematical concepts, Marim and Barbosa [2010] used *GeoGebra* to calculate the necessary measurements of triangular and quadrangular polygons inscribed within the same circle. Measurements were made to determine the best container based on the calculated measures.

GeoGebra was also used by Fernandes and Viseu [2011] to investigate the area of squares inscribed and circumscribed on the same circle. The activity allowed the students to discover for themselves that the more sides the inscribed polygon had; the closer its area was to that of the circle.

3. FINAL CONSIDERATIONS

Rapid technological development means that dynamic and interactive resources are becoming more common in classrooms to assist traditional methods of teaching and learning.

Software interfaces and student-computer interaction vary from simple to more complex. Teachers encounter games, practical exercises and applications. The software can be used as an educational resource to complement classroom content or to introduce mathematical concepts and procedures in the computer lab.

There are many free educational software packages to use in mathematics education. Teachers are faced with choosing the best software among these various options given their teaching and educational needs and the technical requirements of their laboratories.

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