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Mapping free educational software used to develop geometric reasoning

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Abstract

The use of educational software as a teaching resource is becoming more common. However, teachers still need to choose which software best meets needs of their students. The goal of this study was to map free educational software regarding space and shape content for mathematics education. The study was conducted with public elementary teachers (grades 6-9). Of the thirty-three software titles, 55% could contribute to space and shape content. Thirty percent of the software simultaneously developed geometric, numeric and algebraic reasoning as well as metric and proportional reasoning.

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*Keywords:*Free software; grades 6-9; mathematics; geometry;educational software

1. Introduction

Geometry is the study of the properties of and relationships between geometric shapes. Understanding the concepts of space and shape provides a basis for learning mathematics and other subjects (Clements, 1998; Jones, 2002). These concepts are linked to thinking geometrically and developing logical thought. In other words, learning to use geometry is learning to think logically (Surynkova, 2011). By learning geometric concepts, students develop

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special thought processes that allow them to geometrically and systematically understand, describe and represent the world in which they live (Brasil, 1998; National Council of Teachers of Mathematics, 2000). These skills are essential for problem solving.

In order to use geometry to solve problems in mathematics and other fields, students must be able to express problems geometrically (Surynkova,2011). Three registers are possible in geometry: natural language, symbolic language and the figural register (Deliyianni et al.,2009). The figural register depicts relationships between concepts and properties.

There are many cognitive stages ranging from exploring physical objects such as works of art, paintings, drawings, sculptures and crafts (Brazil, 1998), to geometric situations described by natural language and represented by geometric figures.

Several studies have focused on understanding the mental progression of students as they follow these steps (Duval, 1995, 1998, 1999; Fischbein, 1993; Van Hiele, 1986).Our study focuses on the transformation of a real object into a geometric object and the understanding of the object's properties.

In order for students to advance cognitively, teachers use concrete materials (Figueira-Sampaio et al., 2012) and educational software (Figueira-Sampaio et al., 2013).

However, the challenge of choosing and even finding appropriate software may discourage teachers from actually using this educational resource. Therefore, our objective was to present viable software options for developing geometric reasoning by mapping free educational software regarding space and shape content.

2. Methods

We selected 33 software packages from those proposed by Nogueira et al. (2013) and others mentioned in digital or print format. All software packages were free and intended for 6^{th} to 9^{th} grade mathematics education. All of the software packages were easy to install and freely available on the internet.

Thirty-four public middle school teachers (grades 6-9) determined the mathematical content targeted by each software package. To accomplish this, the features of each computer program were presented to the teachers, who then filled out forms identifying the specific mathematical content addressed by each package. The content was organized into thematic blocks based on Brazilian national curriculum parameters (PCNs – Parametros Curriculares Nacionais): Numbers and Operations, Space and Shape, Quantities and Measurements, and Information Processing.

3. Results and Discussion

According to the teachers and based on the thematic blocks (PCNs), 55% of the 33 software packages could be used to work with Space and Shape content. A further 48% of these could also be used for Numbers and Operations and/or Quantities and Measurements content.

The software packages *C.a.R.-Regua e Compasso, Dr Geo, GeoGebra, Geonext, MathGV, Poly* and *Wingeom* were the most frequently identified for the Space and Shape block. These programs address concepts needed to understand displacement and space content and to study geometric shapes (Fig. 1 and 2). Specifically, these programs can be used to work with planar geometric shapes, points, lines and planes; line segments, altitudes, collinear and consecutive line segments; non-planar geometric shapes, vertices, edges and faces; rotations and angles; polygons; points in a Cartesian plane; triangles and their elements; classification of triangles based on sides and angles; quadrangles and their elements; relative positions of two lines in a plane; perimeter and diagonals of polygons; angles of convex and regular polygons.

Of all the teachers who evaluated *Dr Geo, GeoGebra, Geonext, Polígonos, Teorema de Pitagoras, Triangulo, Trigonometria* and *Wingeom* (n=15, n=25, n=24, n=20, n=19, n=18, n=23, n=20, respectively), 100% identified the software as appropriate educational resources for the Space and Shape block. These teachers also indicated that all of these programs could be used to create activities to work with the elements, measurements and classification of angles. Forty-three percent of the professors (n=21) identified *MathGV* as a potential source for developing geometric knowledge. In general, the software could be used to demonstrate and visualize angles, which contributes to the understanding, but not the memorization of content. Understanding content allows students to use concepts to understand and reflect on mathematical problems or concrete cases instead of simply applying rules.

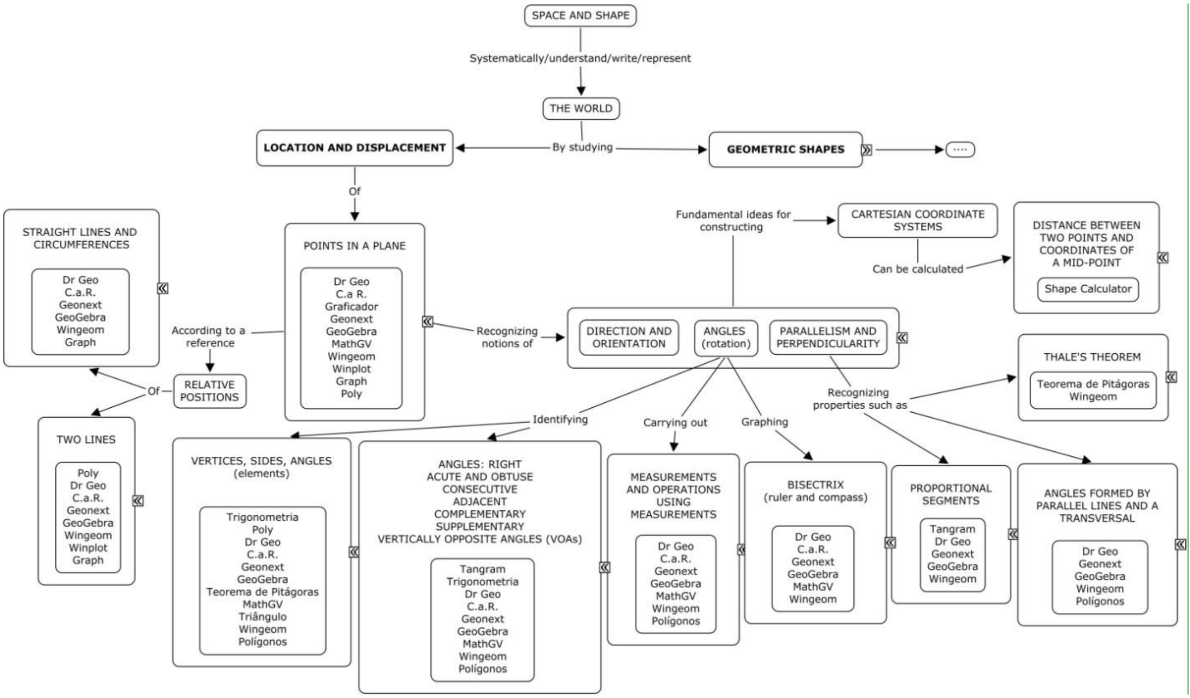

Fig. 1.Software mapping by location and displacement topics.

The study of shapes in elementary school prioritizes the study of polygons and their properties such as the number of sides, length of sides, angle measurements, parallel lines and symmetric lines (Ministry of Education, 2008). However, polygons should not be studied only by themselves. These shapes appear on the faces of objects in the physical world, in works of art, paintings, drawings, sculptures and handcrafts. Thus, the study of polygons should begin with these apparatuses (Brasil, 1998).

Spatial sense is the ability to perceive the environment and the physical objects contained within. This skill is essential for understanding many of the geometric aspects of the world (Ministry of Education, 2005).Objects provide an initial physical model or representation of geometric concepts. For example, these objects allow students to visualize the vertices, faces and edges of a rectangular cuboid package or container. The concrete stage is important, but it should not restrict experimentation. Students must advance cognitively to processes of abstraction, generalization and symbolization and develop ideas of perspective, direction, measurement and location. These advances can be achieved by working with physical materials, two-dimensional models via computer programs and dialectic exploration through verbal language (Clements, 1998). Visualizing, drawing and comparing shapes and figures in various positions contributes to the development of spatial sense (Ministry of Education, 2005).

Tangram 32 was one of the free software packages chosen by the teachers as a potential resource for studying planar geometric shapes in various positions. The program presents two large triangles, three small triangles, one parallelogram and a square that can be combined and organized to form a square. The software allows students to rotate and translate the shapes into differentpositions, contributing to non-stereotypical treatment of geometric objects. *DrGeo, GeoGebra, C.a.R.-Regua e Compasso, Geonext, Graph, Poly, Rompecabezas Pitagoricos* and *Wingeom* can be used in a similar way. Except for *Graph*,the teachers indicated that the remaining software and *Graficador*, *Geometry 2.1*, *Shape Calculator* and *Polígonos* (totaling 13 software packages) could be used to work with polygons (Fig. 2). Not all of these programs could be used for all polygon topics; however, there are still numerous appropriate options regardless of the specific subtopic.

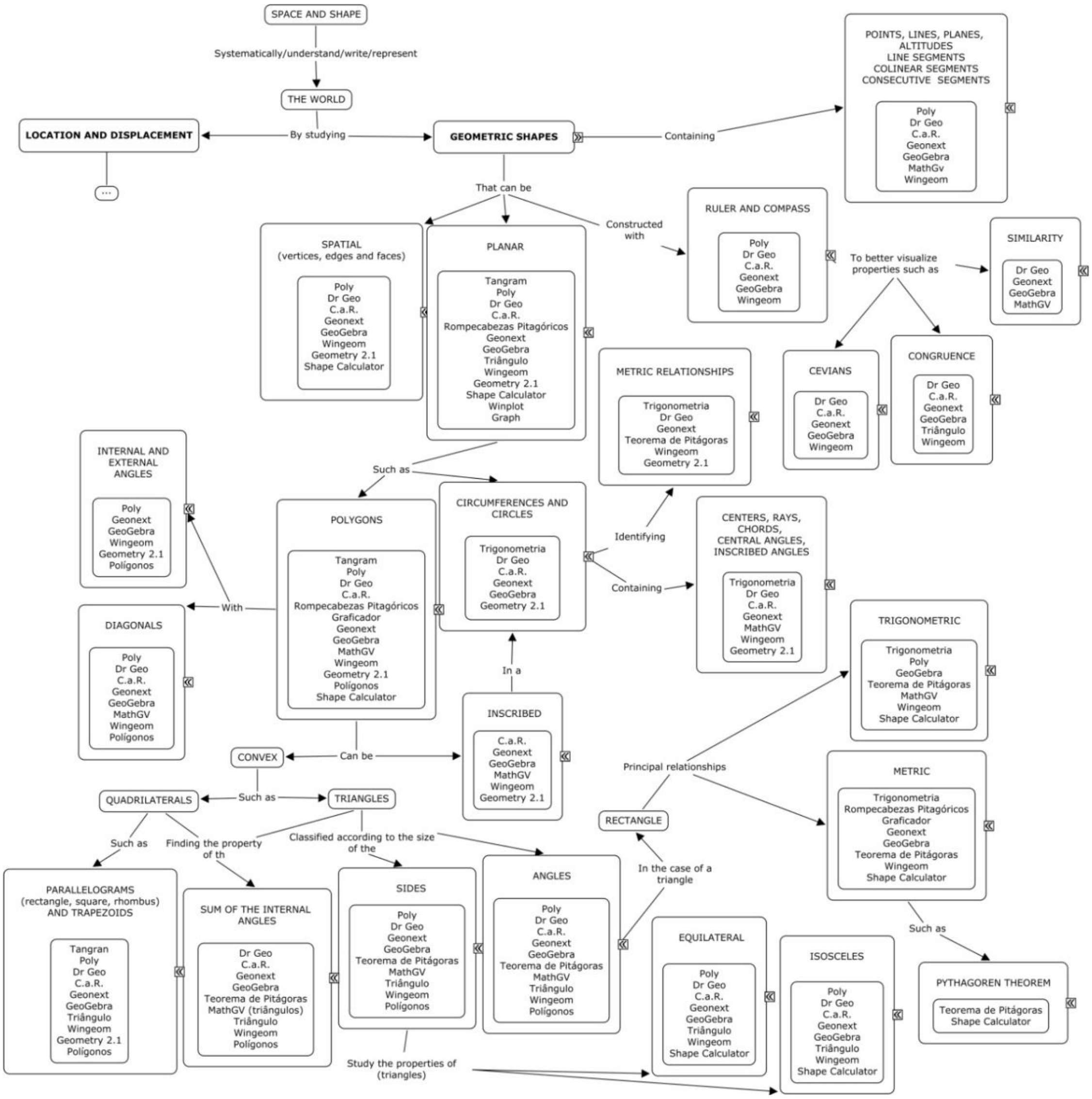

Fig. 2.Software mapping by topic and geometric shape.

Software for Dynamic Geometry such as *C.a.R.-Regua e Compasso, Geonext, GeoGebra,* and *Dr Geo*, allows teachers to not only rotate and translate polygons, but also create exercises based on the construction of polygons. Therefore, these exercises can also be used to explore elements of polygons such as sides, vertices, angles and their properties.

Software packages targeted at Dynamic Geometry have features that allow the construction of geometric objects through experience and based on visual perception or through geometric control where the objects can be designed from definable properties. Visual perception can be used to construct a rectangle by creating its vertices over a checkered grid. The "move" function can then be used to move the vertices and deform the rectangle into another shape such as a trapezoid. Geometric control allows the user to produce a shape similar to that created by visual perception. With geometric controls, students use the properties of a rectangle (90º angles, opposing parallel sides) to create the rectangle, but are unable to deform the shape.

In Dynamic Geometry, base points can be freely constructed on a grid or curve and then freely dragged to different locations. Dependent points, those whose existence depends on other elements, cannot be dragged directly (Baccaglini-Frank, Mariotti & Antonini, 2009). In constructions made from their defining properties, the dynamic aspect of the software allows students to drag the base points and depict geometric invariants (Laborde, 2005). Dynamic behavior refers to the degree of freedom available for moveable elements and the response of other elements relative to these changes and invariants (Talmon & Yerushalmy, 2004, 2006). Therefore, designs that can be dragged are not only designs that can be moved and changed, but a sequence of different designs with certain common properties (Hölzl, 2001) or invariants. Talmon and Yerushalmy (2004) provide an example of dynamic behavior in the construction of parallelograms. The authors highlight the dynamic behavior allowed by the software while producing different families of parallelograms and consequently evidencing the invariants.

Invariants help students discover many geometric properties while moving objects. Properties such as the congruence and similarity of geometric shapes and Cevians can be studied through geometric control. The "move" function in software allows students to drag some elements while observing the resulting sequence of designs as well as the similarities, differences and regularities.

Auxiliary constructions are needed to construct geometric shapes. The final shape depends on these elements; however, these elements distract student attention from the process of experimentation (Olivero, 2005) and hinder the visualization of the properties that are the target of the investigation. Many examples of Dynamic Geometry software allow users to "hide" auxiliary elements without deleting them completely. Some elements cannot be deleted without deleting the final figure. Olivero (2005) highlights the possibility of using the "hide" function to hide and display auxiliary elements when necessary.

Teachers can use the "show/hide" and "activate/disactivate" functions in *Geogebra* and *Geonext*, respectively. *C.a.R.-Regua* and *Compasso* can also hide auxiliary elements, but in different ways. Two functions in the toolbar allow the user to "conceal objects" and "reveal concealed objects". The "reveal" function shows all concealed elements at the same time. Consequently, teachers cannot reveal objects individually to promote student understanding. The same function in *Dr. Geo* excludes objects instead of hiding them, making it impossible to re-exhibit them later.

The *Poligonos* and *Shape Calculator* software packages provide alternate ways of working with polygons. After constructing necessary concepts, the teacher can then use the software in non-investigative ways. These programs can be used in conjunction with other software or educational resources in constructive ways.

The interface of the *Poligonos* software shows a text box where students can enter a desired number of sides for a polygon. The software then returns the name of the polygon, the sum of its internal and external angles, the internal and external angles and the number and type of diagonals. The software simply shows the main characteristics of a polygon. *Shape Calculator* provides various options for different geometric shapes. Choosing a specific polygon opens a second window. If a triangle is chosen for example, an image of a triangle appears with fields where students can enter the values for the sides, base and height of the triangle. The software then calculates the area and perimeter of the triangle. This provides teachers with the ability to explore the specific characteristics of isosceles, equilateral and even scalene triangles by changing the "side" values. Nevertheless, the software only shows a symbolic representation of the shape, not the actual shape based on the values entered.

Understanding polyhedra and their properties helps students make connections between two and three-dimensional geometry (Ministry of Education, 2008). To describe an object in two dimensions, students describe some of its characteristics, establish relationships between the characteristics and use correct nomenclature (Brasil, 1998).

Poly was the software most recommended by the teachers for activities with three-dimensional shapes. This program allows students to transfer two-dimensional shapes to three dimensions and vice versa. The software interface shows a solid that can be transformed step-by-step into a two-dimensional shape by sliding a scroll bar. Students can then explore each two-dimensional face of the solid. Sliding the scroll bar also allows students to see the geometric properties such as vertices, faces and edges, of the three-dimensional format.

Wingeom, Geometry 2.1 and *Shape Calculator* can be used to work with shapes in two and three dimensions. Thus, the teachers also indicated that these programs could be used to work with three-dimensional shapes. *Geometry 2.1* and *Shape Calculator* employ a different investigative approach by working with an image, its name and the formula of the three-dimensional shape. These programs are targeted at working with formulas.

Middle school students should have the flexibility to switch between two and three-dimensional representations of a shape. Two-dimensional representations can include networked-two-dimensional images showing each face of a three-dimensional shape. By working with these networks, students develop the spatial awareness needed to connect or form the two-dimensional images into a three-dimensional shape (Ministry of Education, 2008). To accomplish this thought process, students must be able to construct the three-dimensional shape by folding. Even though *Dr Geo, C.a.R.-Regua e Compasso, Geonext* and *GeoGebra* are targeted at planar shapes, some of the teachers thought they could still be used to construct the networks of three-dimensional shapes.

The study of space and shape also includes locating and displacing points in a plane (Brasil, 1998; National Council of Teachers of Mathematics, 2000). To begin to understand the spatial relationships of a point, it is first necessary to associate this point to both a horizontal and a vertical position (Ministry of Education, 2008).

GeoGebra, Geonext, Dr Geo, Winplot, Graph, MathGV and *Wingeom* were chosen by the teachers for creating exercises on the displacement of points in a plane (Fig. 1). These programs allow users to place coordinate axes on a virtual desktop. *GeoGebra, Winplot, Wingeom* and *MathGV* show only one of the axes, which are useful for working with the location of points. *Graficador* shows fixed coordinate axes that cannot be hidden and then redisplayed.

The systematic representation of conventional mathematical objects such as points, lines and curves within a reference system (Brasil, 1998), including cartesian planes, is essential for continued learning in geometry and algebra (Ministry of Education, 2008).The teachers recommended *GeoGebra, Geonext, Dr Geo, Winplot, Graph* and *Wingeom* for working with point location and displacement. All of these programs provide a checkered grid with coordinate axes to help students understand the location of points by ordered pairs. In *Dr Geo*, the checkered grid is tied to the axes and neither the grid nor the axes can be hidden

4. Conclusion

Our software maps show numerous free software options that teachers can use to help build an understanding of geometry. These educational resources help both students and teachers. For teachers, these tools supply means to visually demonstrate concepts and properties. They also make it easier to bring together school activities and the daily realities of students. Outside of school, students use these resources daily. For students, these tools make the educational process easier, more interactive and more enjoyable.

Teachers must analyze the characteristics of each software package. Interfaces vary from simple to more complex and teachers can choose software with both dynamic and static content. Some software requires students to take a more investigative approach. Other software types use the direct application of formulas, which implies that teachers have already explored the content and constructed concepts via other software or resources. Using more than one software package for the same educational objective means that the programs may compensate for each other's limitations.

When the software map shows more than one software option for a specific educational objective, the final choice should be based on the teaching-learning requirements of the teacher and the technical requirements of the computer laboratory and the educational institution.

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